In Class Exercises Set A

Answers and Explanations: Math Algebra 2

1) The answer is **A)**. $f(x-h)+k$ is translated h units to the right and k units up from $f(x)$. The transformation can be rewritten as $f(x-(-3))+2$; hence, the point will be translated 3 units to the left and two units up to the point $(5, 2)$. (1)

2) The answer is **A)**. The quadratic factors into $(x-5)(x-4)=0$, giving us two positive roots at 5 and 4. A faster solution can come from knowing that the general form of a quadratic function with two positive zeroes will be $a(x-b)(x-c)$, where b and c are positive numbers. When expanded, the coefficient on x must be negative while the constant must be positive, and A) is the only one that fits. (1)

3) The answer is **D)**. To simplify a fraction including either irrationals or imaginary numbers, multiply the numerator and denominator by the conjugate pair of the denominator. In this case, the conjugate of $6+2i$ is $6-2i$. From there, utilize foil: the numerator becomes $[(5)(6)+(-i)(6)+(5)(-2i)+(-i)(-2i)] = 28-16i$ (Note that $i^2=-1$). The denominator is a difference of two squares: $6^2-(2i)^2=40$. This fraction reduces to the answer, $\frac{7-4i}{10}$. (2)

4) The answer is **A)**. The amount of money that Santiago can make is equal to the number of donuts that he sells times the price of each donut. In cents, this is $p(100-p) = 100p - p^2$. Recognize that this is a down-opening quadratic function: meaning that its maximum occurs at its vertex. The x-coordinate of the vertex is given by $-\frac{b}{2a} = -\frac{100}{2(-1)} = 50$. If he sells the donuts for 50 cents each, his gains will total $(50)(100-50)$= 2500 cents or \$25.00. (2)

5) The answer is **C)**. It doesn't matter that a, b, and c are positive: any quadratic function is even across its axis of symmetry. The x-value of the axis of symmetry is equal to $-\frac{b}{2a}$, so it must be true that an x coordinate any number of units to the left of it has the same value for $f(x)$ as a coordinate that many units to the right. This is equivalent to answer choice C). (2)

6) The answer is **12**. If $x=3$ is a root, then the function must divide the term $x-3$ evenly (with no remainder). Either synthetic or long division shows that the remainder when dividing by -3 is $-k+12$, which is necessarily 0, so $k=12$. Slightly more elegantly, you could also plug 3 into the function, and find that $f(3)=-k+12=0$. Note: these two methods prove the Remainder Theorem, or $R[f(x)/(x-c)]=f(c)$. (2)

7) The answer is **1**. First apply the transformation: $g(x+2)+6 = 2(x+2) - 6 + 6 = 2x + 4$. Next, set the functions equal and solve: $x^2+5 = 2x+4$; $x^2-2x+1 = 0$; $(x-1)^2=0$. Thus, $x=1$ is the x-coordinate of the only solution. (3)

8) The answer is **3** seconds. Don't be phased by the constant k. Begin by setting the equation equal to a quarter of the initial height (aka the y position at $t=0$, plugging in shows that this is $4k$), leaving you with k = $-kt^2+2kt+4k$. Getting everything on one side, $kt^2-2kt-3k=0$. This neatly factors into k(t – 3)(t + 1). The event begins at t = 0, so the answer must be positive: t = 3. (Life Pro Tip: the answers on open-ended SAT grid-ins can neither be negative nor occupy more than 4 boxes when written as decimals/fractions). (3)